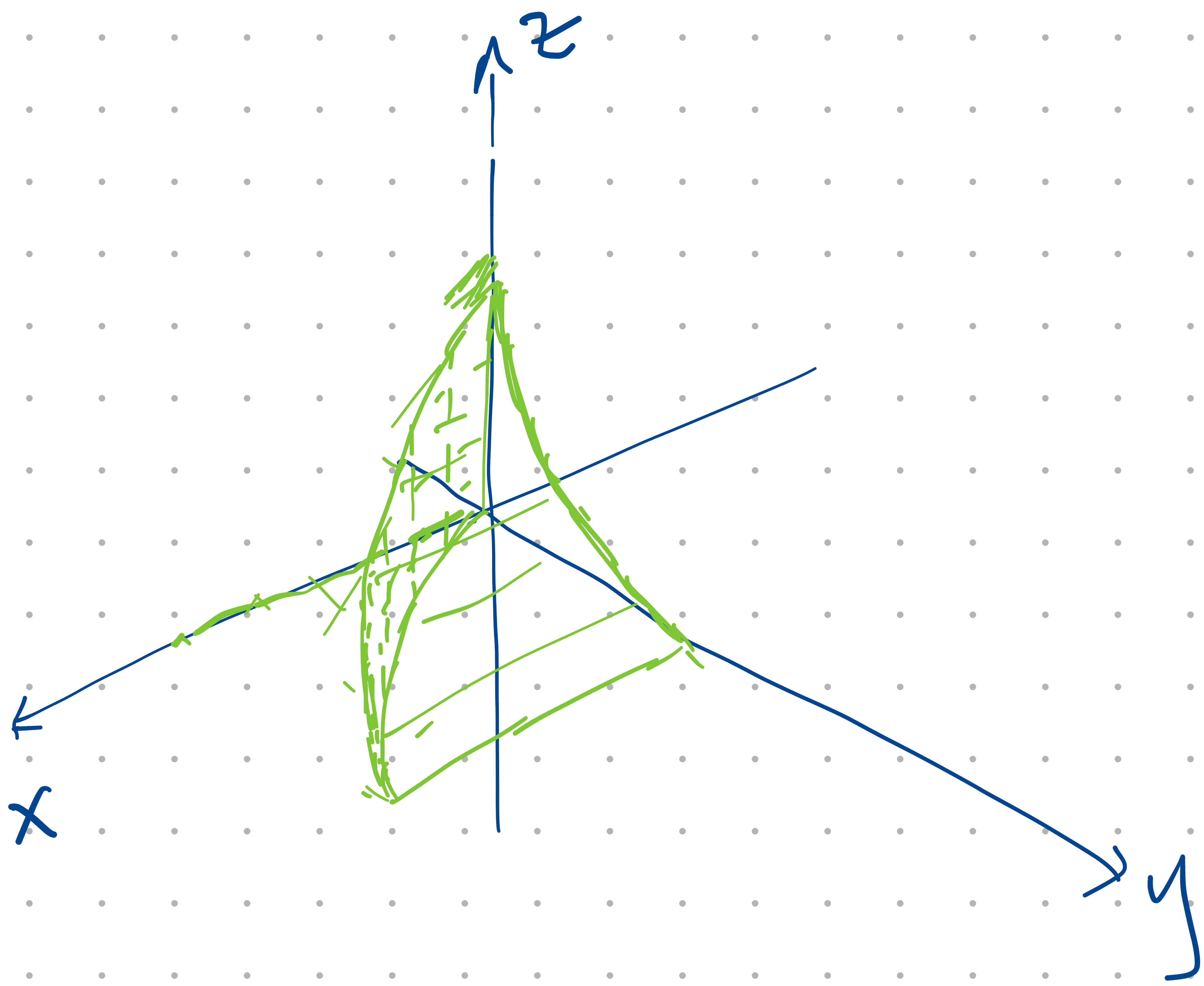
1) Express the below integral in the integration orders

a) $dz\,dx\,dy$ and b) $dx\,dz\,dy$.

$$\int_0^1 \int_{x^2}^1 \int_0^{1-\sqrt{y}} f(x,y,z)\, dz\, dy\, dx$$



2) An icecream cone (filled w/ ice cream) consists of the solid region between

$$z = 2\sqrt{x^2+y^2} \quad \text{and} \quad z = \sqrt{5-x^2-y^2}$$

Setup a triple integral which computes its volume in

a) Cylindrical

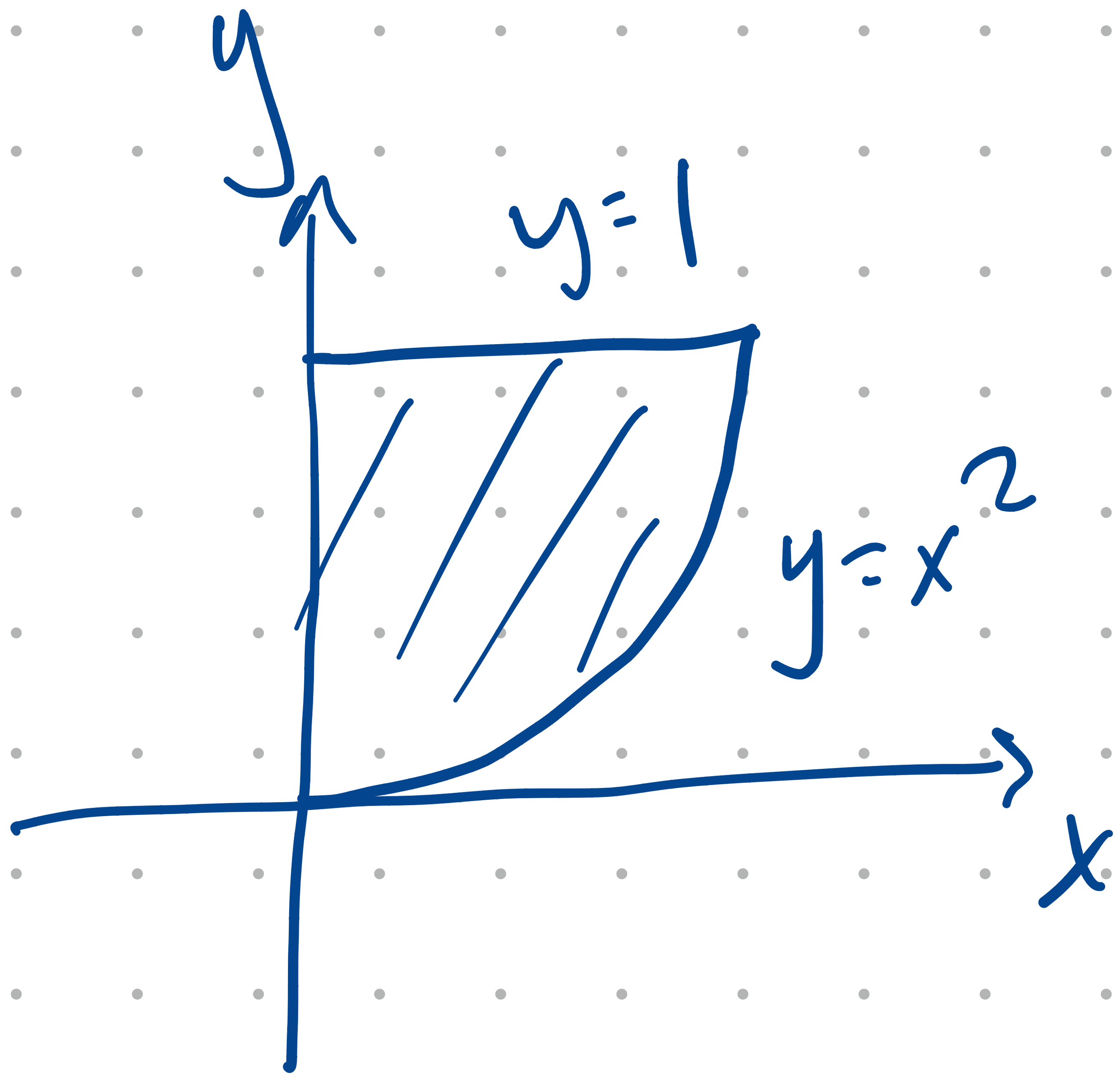
b) Spherical

Then, evaluate whichever of the two integrals seems better.

a) $$\int_0^1 \int_{x^2}^1 \boxed{\int_0^{1-\sqrt{y}} f(x,y,z)\,dz}\,dy\,dx$$

some expression $g(x,y)$

$$\int_0^1 \int_{x^2}^1 g(x,y)\,dy\,dx$$



$$= \int_0^1 \int_0^{\sqrt{y}} g(x,y)\,dx\,dy$$

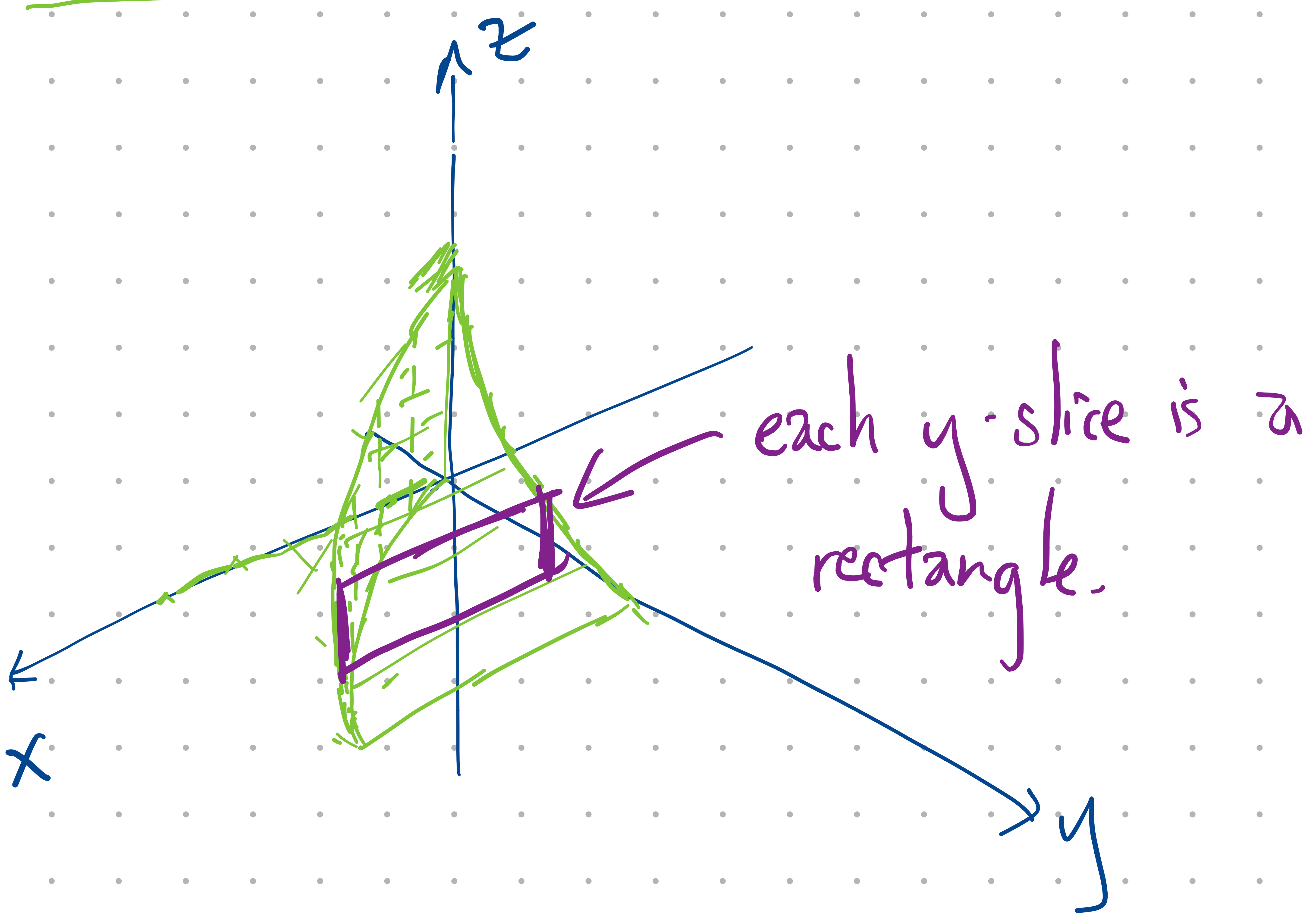
$$= \int_0^1 \int_0^{\sqrt{y}} \int_0^{1-\sqrt{y}} f(x,y,z)\,dz\,dx\,dy$$

b) $$\int_0^1 \boxed{\int_0^{\sqrt{y}} \int_0^{1-\sqrt{y}} f(x,y,z)\,dz\,dx}\,dy$$

$\Vert$ Fubini

$\leftarrow$ y is a constant for the purposes of this double integral

$$\int_0^1 \boxed{\int_0^{1-\sqrt{y}} \int_0^{\sqrt{y}} f(x,y,z)\,dx\,dz}\,dy$$



Alternate (direct) method for b)

$\boxed{0 \le x \le 1}$ $\boxed{0 \le \sqrt{y}}$

$\boxed{x^2 \le \boxed{y} \le 1}$ $\boxed{0 \le 1-\sqrt{y}}$

$\boxed{0 \le z \le 1-\sqrt{y}}$

$\underline{\underline{dx}}\,dz\,dy$: Inequalities involving x

$0 \le x \le 1$ — redundant b/c of $\Downarrow$

and $x \le \sqrt{y} \le 1$

So x bounds are $\underline{0 \text{ to } \sqrt{y}}$.

$dx\,\underline{\underline{dz}}\,dy$: Remaining ineqs involving z.

$0 \le z \le 1-\sqrt{y}$

So z bounds $\underline{0 \text{ to } 1-\sqrt{y}}$

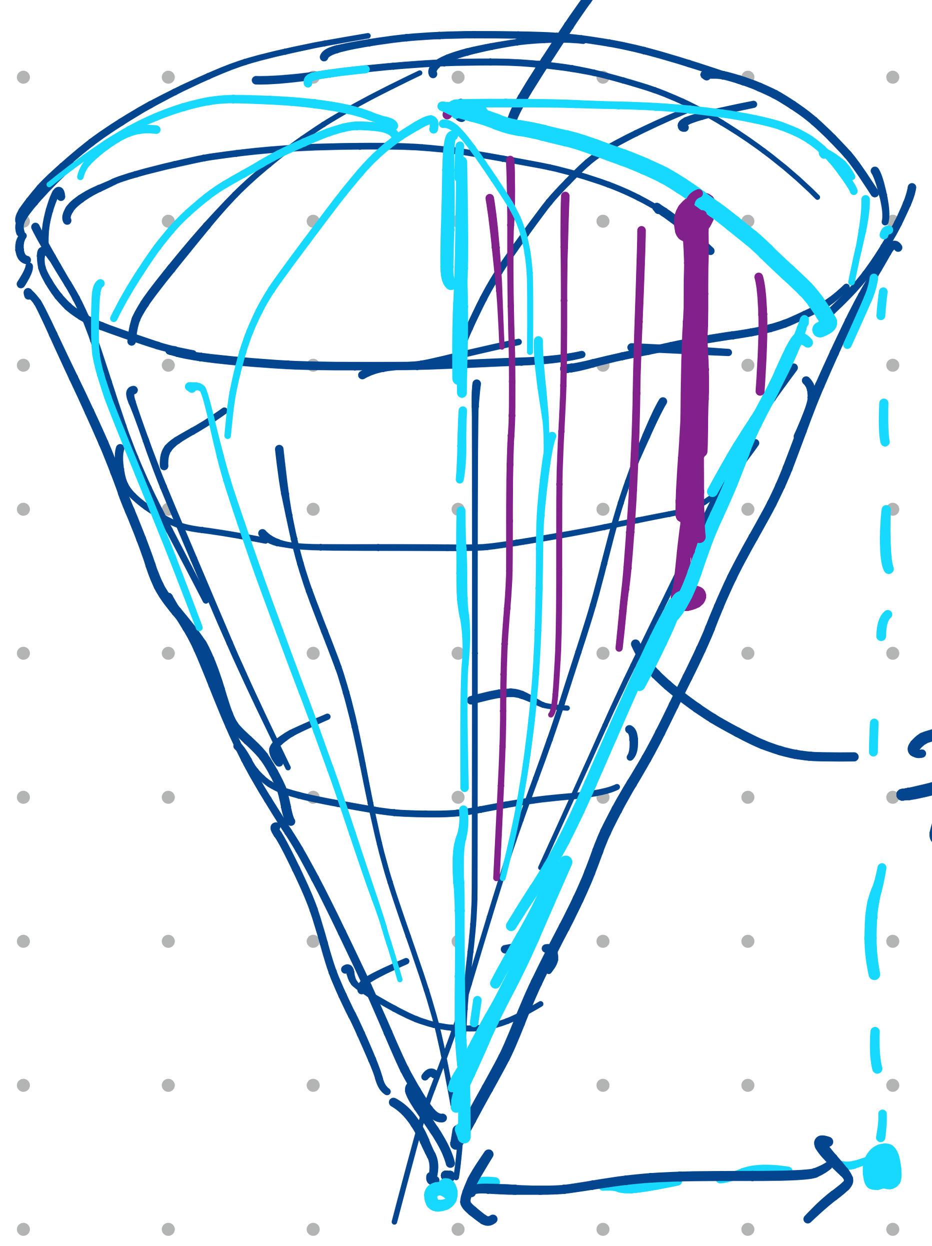
dxdz$\underline{dy}$: remaining ineqs involving y

conclude:

$$0 \le y \le 1$$

$$\int_0^1 \int_0^{1-\sqrt{y}} \int_0^{\sqrt{y}} \text{\_\_\_\_} \, dx\, dz\, dy$$

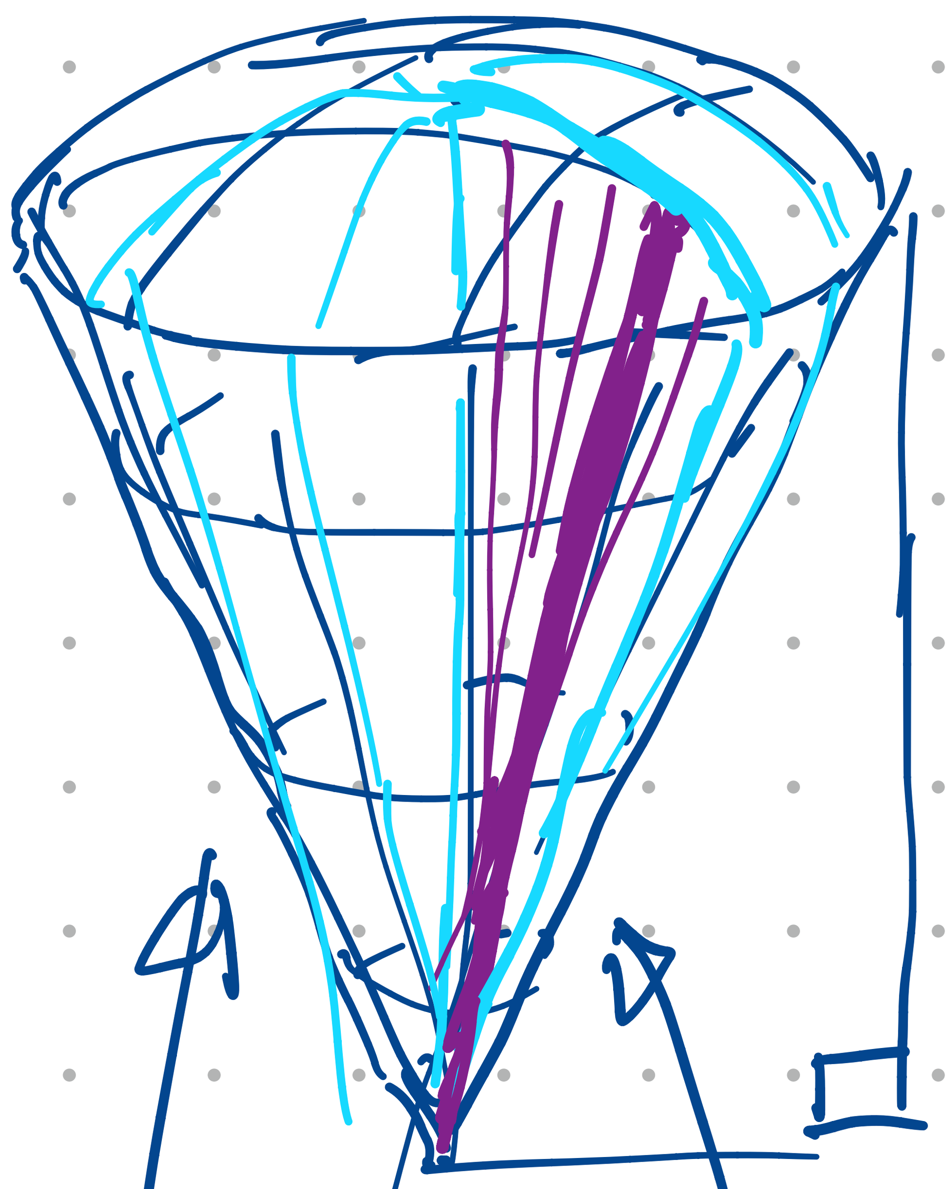
2) a)



$$\int_0^{2\pi}\int_0^{1}\int_{2r}^{\sqrt{5-r^2}} 1 \; r\,dz\,dr\,d\theta$$

1 from $2r=\sqrt{5-r^2}$

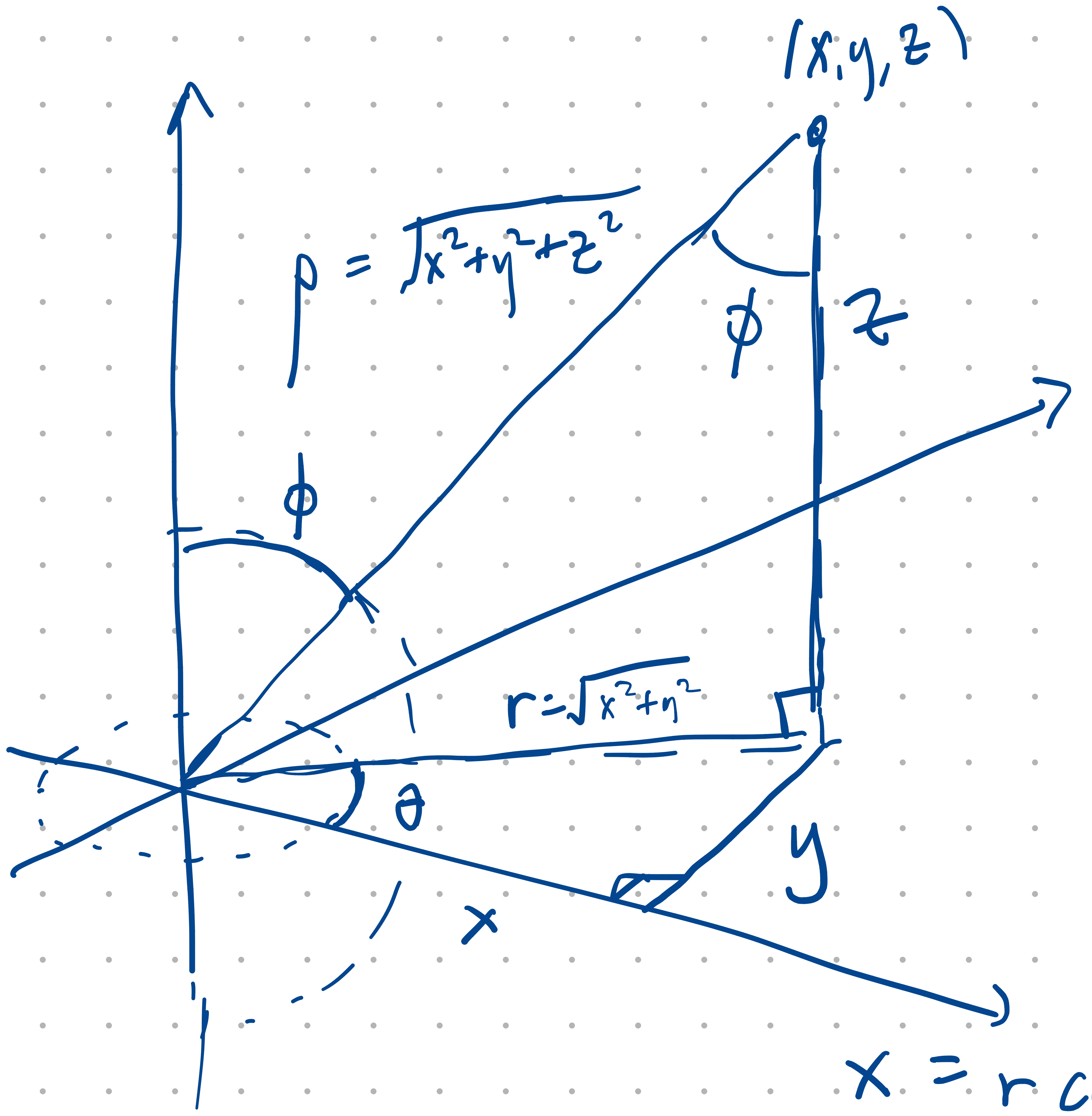
$z^2=5-x^2-y^2$, $\quad x^2+y^2+z^2=5$, $\quad \boxed{\rho^2=5}$



$$\int_0^{2\pi}\int_0^{\arctan(1/2)}\int_0^{\sqrt{5}} 1 \; \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta \;!$$

slope 2.

$z=2\sqrt{x^2+y^2}$, $\quad \dfrac{r}{z}=\dfrac{1}{2}$, $\quad \boxed{\tan\phi = 1/2}$



$x = r\cos\theta$

$= \rho\sin\phi\cos\theta$

$\tan\phi = \dfrac{r}{z}$

$y = r\sin\theta$

$\sin\phi = r/\rho$

$= \rho\sin\phi\sin\theta$

$\cos\phi = z/\rho$

$z = r\cos\phi$